

# Funcons-beta: Binding \*

The P<sub>L</sub>anC<sub>o</sub>mpS Project

Binding.cbs | PLAIN | PRETTY

## OUTLINE

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## Binding

- [ *Type* environments
- Alias* envs
- Datatype* identifiers
- Alias* ids
- Funcon* identifier-tagged
- Alias* id-tagged
- Funcon* fresh-identifier
- Entity* environment
- Alias* env
- Funcon* initialise-binding
- Funcon* bind-value
- Alias* bind
- Funcon* unbind
- Funcon* bound-directly
- Funcon* bound-value
- Alias* bound
- Funcon* closed
- Funcon* scope
- Funcon* accumulate
- Funcon* collateral
- Funcon* bind-recursively
- Funcon* recursive ]

*Meta-variables*  $T <: \text{values}$

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\*Suggestions for improvement: [plancomps@gmail.com](mailto:plancomps@gmail.com).  
Reports of issues: <https://github.com/plancomps/CBS-beta/issues>.

## Environments

*Type* `environments`  $\rightsquigarrow$  `maps(identifiers, values?)`

*Alias* `envs` = `environments`

An environment represents bindings of identifiers to values. Mapping an identifier to `( )` represents that its binding is hidden.

Circularity in environments (due to recursive bindings) is represented using bindings to cut-points called `links`. Funcons are provided for making declarations recursive and for referring to bound values without explicit mention of links, so their existence can generally be ignored.

*Datatype* `identifiers` ::= `{_ : strings}` | `identifier-tagged(_ : identifiers, _ : values)`

*Alias* `ids` = `identifiers`

*Alias* `id-tagged` = `identifier-tagged`

An identifier is either a string of characters, or an identifier tagged with some value (e.g., with the identifier of a namespace).

*Funcon* `fresh-identifier` :  $\Rightarrow$  `identifiers`

`fresh-identifier` computes an identifier distinct from all previously computed identifiers.

*Rule* `fresh-identifier`  $\rightsquigarrow$  `identifier-tagged("generated", fresh-atom)`

## Current bindings

*Entity* `environment(_ : environments)`  $\vdash$  `_`  $\longrightarrow$  `_`

*Alias* `env` = `environment`

The environment entity allows a computation to refer to the current bindings of identifiers to values.

*Funcon* `initialise-binding(X :  $\Rightarrow$  T)` :  $\Rightarrow$  `T`  
 $\rightsquigarrow$  `initialise-linking(initialise-generating(closed(X)))`

`initialise-binding(X)` ensures that `X` does not depend on non-local bindings. It also ensures that the linking entity (used to represent potentially cyclic bindings) and the generating entity (for creating fresh identifiers) are initialised.

*Funcon* `bind-value(I : identifiers, V : values)` :  $\Rightarrow$  `environments`  
 $\rightsquigarrow$  `{I  $\mapsto$  V}`

*Alias* `bind` = `bind-value`

`bind-value(I, X)` computes the environment that binds only `I` to the value computed by `X`.

*Funcon* `unbind(I : identifiers)` :  $\Rightarrow$  `environments`  
 $\rightsquigarrow$  `{I  $\mapsto$  ( )}`

`unbind(I)` computes the environment that hides the binding of `I`.

*Funcon* `bound-directly`(`_ : identifiers`) :  $\Rightarrow$  `values`

`bound-directly`(`l`) returns the value to which `l` is currently bound, if any, and otherwise fails.

`bound-directly`(`l`) does *not* follow links. It is used only in connection with recursively-bound values when references are not encapsulated in abstractions.

*Rule* 
$$\frac{\text{lookup}(\rho, l) \rightsquigarrow (V : \text{values})}{\text{environment}(\rho) \vdash \text{bound-directly}(l : \text{identifiers}) \longrightarrow V}$$

*Rule* 
$$\frac{\text{lookup}(\rho, l) \rightsquigarrow ()}{\text{environment}(\rho) \vdash \text{bound-directly}(l : \text{identifiers}) \longrightarrow \text{fail}}$$

*Funcon* `bound-value`(`l : identifiers`) :  $\Rightarrow$  `values`  
 $\rightsquigarrow$  `follow-if-link`(`bound-directly`(`l`))

*Alias* `bound` = `bound-value`

`bound-value`(`l`) inspects the value to which `l` is currently bound, if any, and otherwise fails. If the value is a link, `bound-value`(`l`) returns the value obtained by following the link, if any, and otherwise fails. If the inspected value is not a link, `bound-value`(`l`) returns it.

`bound-value`(`l`) is used for references to non-recursive bindings and to recursively-bound values when references are encapsulated in abstractions.

## Scope

*Funcon* `closed`(`X :  $\Rightarrow$  T`) :  $\Rightarrow$  `T`

`closed`(`X`) ensures that `X` does not depend on non-local bindings.

*Rule* 
$$\frac{\text{environment}(\text{map}(\ )) \vdash X \longrightarrow X'}{\text{environment}(\_) \vdash \text{closed}(X) \longrightarrow \text{closed}(X')}$$

*Rule* `closed`(`V : T`)  $\rightsquigarrow$  `V`

*Funcon* `scope`(`_ : environments, _ :  $\Rightarrow$  T`) :  $\Rightarrow$  `T`

`scope`(`D, X`) executes `D` with the current bindings, to compute an environment  $\rho$  representing local bindings. It then executes `X` to compute the result, with the current bindings extended by  $\rho$ , which may shadow or hide previous bindings.

`closed`(`scope`( $\rho, X$ )) ensures that `X` can reference only the bindings provided by  $\rho$ .

*Rule* 
$$\frac{\text{environment}(\text{map-override}(\rho_1, \rho_0)) \vdash X \longrightarrow X'}{\text{environment}(\rho_0) \vdash \text{scope}(\rho_1 : \text{environments}, X) \longrightarrow \text{scope}(\rho_1, X')}$$

*Rule* `scope`(`_ : environments, V : T`)  $\rightsquigarrow$  `V`

*Funcon* `accumulate`(`_ : ( $\Rightarrow$  environments)*`) :  $\Rightarrow$  `environments`

`accumulate`(`D1, D2`) executes `D1` with the current bindings, to compute an environment  $\rho_1$  representing some local bindings. It then executes `D2` to compute an environment  $\rho_2$  representing further local bindings, with the current bindings extended by  $\rho_1$ , which may shadow or hide previous current bindings. The result is  $\rho_1$  extended by  $\rho_2$ , which may shadow or hide the bindings of  $\rho_1$ .

`accumulate`(`_, _`) is associative, with `map`(`_`) as unit, and extends to any number of arguments.

$$\text{Rule } \frac{D_1 \longrightarrow D'_1}{\text{accumulate}(D_1, D_2) \longrightarrow \text{accumulate}(D'_1, D_2)}$$

$$\text{Rule } \text{accumulate}(\rho_1 : \text{environments}, D_2) \rightsquigarrow \text{scope}(\rho_1, \text{map-override}(D_2, \rho_1))$$

$$\text{Rule } \text{accumulate}(\ ) \rightsquigarrow \text{map}(\ )$$

$$\text{Rule } \text{accumulate}(D_1) \rightsquigarrow D_1$$

$$\text{Rule } \text{accumulate}(D_1, D_2, D^+) \rightsquigarrow \text{accumulate}(D_1, \text{accumulate}(D_2, D^+))$$

$$\begin{aligned} \text{Funcon } \text{collateral}(\rho^* : \text{environments}^*) : &\Rightarrow \text{environments} \\ &\rightsquigarrow \text{checked map-unite}(\rho^*) \end{aligned}$$

$\text{collateral}(D_1, \dots)$  pre-evaluates its arguments with the current bindings, and unites the resulting maps, which fails if the domains are not pairwise disjoint.

$\text{collateral}(D_1, D_2)$  is associative and commutative with  $\text{map}(\ )$  as unit, and extends to any number of arguments.

## Recurse

$$\begin{aligned} \text{Funcon } \text{bind-recursively}(I : \text{identifiers}, E : &\Rightarrow \text{values}) : \Rightarrow \text{environments} \\ &\rightsquigarrow \text{recursive}(\{I\}, \text{bind-value}(I, E)) \end{aligned}$$

$\text{bind-recursively}(I, E)$  binds  $I$  to a link that refers to the value of  $E$ , representing a recursive binding of  $I$  to the value of  $E$ . Since  $\text{bound-value}(I)$  follows links, it should not be executed during the evaluation of  $E$ .

$$\begin{aligned} \text{Funcon } \text{recursive}(SI : \text{sets}(\text{identifiers}), D : &\Rightarrow \text{environments}) : \Rightarrow \text{environments} \\ &\rightsquigarrow \text{re-close}(\text{bind-to-forward-links}(SI), D) \end{aligned}$$

$\text{recursive}(SI, D)$  executes  $D$  with potential recursion on the bindings of the identifiers in the set  $SI$  (which need not be the same as the set of identifiers bound by  $D$ ).

$$\begin{aligned} \text{Auxiliary Funcon } \text{re-close}(M : \text{maps}(\text{identifiers}, &\text{links}), D : \Rightarrow \text{environments}) : \Rightarrow \text{environments} \\ &\rightsquigarrow \text{accumulate}(\text{scope}(M, D), \text{sequential}(\text{set-forward-links}(M), \text{map}(\ ))) \end{aligned}$$

$\text{re-close}(M, D)$  first executes  $D$  in the scope  $M$ , which maps identifiers to freshly allocated links. This computes an environment  $\rho$  where the bound values may contain links, or implicit references to links in abstraction values. It then sets the link for each identifier in the domain of  $M$  to refer to its bound value in  $\rho$ , and returns  $\rho$  as the result.

$$\begin{aligned} \text{Auxiliary Funcon } \text{bind-to-forward-links}(SI : \text{sets}(\text{identifiers})) : &\Rightarrow \text{maps}(\text{identifiers}, \text{links}) \\ &\rightsquigarrow \text{map-unite}(\text{interleave-map}(\text{bind-value}(\text{given}, \text{fresh-link}(\text{values})), \\ &\text{set-elements}(SI))) \end{aligned}$$

$\text{bind-to-forward-links}(SI)$  binds each identifier in the set  $SI$  to a freshly allocated link.

$$\begin{aligned} \text{Auxiliary Funcon } \text{set-forward-links}(M : \text{maps}(\text{identifiers}, &\text{links})) : \Rightarrow \text{null-type} \\ &\rightsquigarrow \text{effect}(\text{interleave-map}(\text{set-link}(\text{map-lookup}(M, \text{given}), \text{bound-value}(\text{given})), \\ &\text{set-elements}(\text{map-domain}(M)))) \end{aligned}$$

For each identifier  $I$  in the domain of  $M$ ,  $\text{set-forward-links}(M)$  sets the link to which  $I$  is mapped by  $M$  to the current bound value of  $I$ .