

Funcons-beta: Bits *

The PLanCompS Project

Bits.cbs | PLAIN | PRETTY

OUTLINE

Bits and bit vectors
 Bits
 Bit vectors

Bits and bit vectors

```
[ Type bits
  Datatype bit-vectors
  Funcon bit-vector
    Type bytes
    Alias octets
  Funcon bit-vector-not
  Funcon bit-vector-and
  Funcon bit-vector-or
  Funcon bit-vector-xor
  Funcon bit-vector-shift-left
  Funcon bit-vector-logical-shift-right
  Funcon bit-vector-arithmetic-shift-right
  Funcon integer-to-bit-vector
  Funcon bit-vector-to-integer
  Funcon bit-vector-to-natural
  Funcon unsigned-bit-vector-maximum
  Funcon signed-bit-vector-maximum
  Funcon signed-bit-vector-minimum
  Funcon is-in-signed-bit-vector
  Funcon is-in-unsigned-bit-vector ]
```

Bits

Type bits \rightsquigarrow booleans

false represents the absence of a bit, true its presence.

*Suggestions for improvement: plancomps@gmail.com.
Reports of issues: <https://github.com/plancomps/CBS-beta/issues>.

Bit vectors

Datatype $\text{bit-vectors}(N : \text{natural-numbers}) ::= \text{bit-vector}(_ : \text{bits}^N)$

Type $\text{bytes} \rightsquigarrow \text{bit-vectors}(8)$

Alias $\text{octets} = \text{bytes}$

Meta-variables $BT <: \text{bit-vectors}(_)$

Built-in Funcon $\text{bit-vector-not}(_ : BT) : \Rightarrow BT$

Built-in Funcon $\text{bit-vector-and}(_ : BT, _ : BT) : \Rightarrow BT$

Built-in Funcon $\text{bit-vector-or}(_ : BT, _ : BT) : \Rightarrow BT$

Built-in Funcon $\text{bit-vector-xor}(_ : BT, _ : BT) : \Rightarrow BT$

The above four funcons are the natural extensions of funcons from `booleans` to `bit-vectors(N)` of the same length.

Built-in Funcon $\text{bit-vector-shift-left}(_ : BT, _ : \text{natural-numbers}) : BT$

Built-in Funcon $\text{bit-vector-logical-shift-right}(_ : BT, _ : \text{natural-numbers}) : BT$

Built-in Funcon $\text{bit-vector-arithmetic-shift-right}(_ : BT, _ : \text{natural-numbers}) : BT$

Built-in Funcon $\text{integer-to-bit-vector}(_ : \text{integers}, N : \text{natural-numbers}) : \text{bit-vectors}(N)$

`integer-to-bit-vector(M, N)` converts an integer M to a bit-vector of length N , using Two's Complement representation. If the integer is out of range of the representation, it will wrap around (modulo 2^N).

Built-in Funcon $\text{bit-vector-to-integer}(_ : BT) : \Rightarrow \text{integers}$

`bit-vector-to-integer(B)` interprets a bit-vector BV as an integer in Two's Complement representation.

Built-in Funcon $\text{bit-vector-to-natural}(_ : BT) : \Rightarrow \text{natural-numbers}$

`bit-vector-to-natural(BV)` interprets a bit-vector BV as a natural number in unsigned representation.

Funcon $\text{unsigned-bit-vector-maximum}(N : \text{natural-numbers}) : \Rightarrow \text{natural-numbers}$
 $\rightsquigarrow \text{integer-subtract}(\text{integer-power}(2, N), 1)$

Funcon $\text{signed-bit-vector-maximum}(N : \text{natural-numbers}) : \Rightarrow \text{integers}$
 $\rightsquigarrow \text{integer-subtract}(\text{integer-power}(2, \text{integer-subtract}(N, 1)), 1)$

Funcon $\text{signed-bit-vector-minimum}(N : \text{natural-numbers}) : \Rightarrow \text{integers}$
 $\rightsquigarrow \text{integer-negate}(\text{integer-power}(2, \text{integer-subtract}(N, 1)))$

Funcon $\text{is-in-signed-bit-vector}(M : \text{integers}, N : \text{natural-numbers}) : \Rightarrow \text{booleans}$
~~~ $\rightsquigarrow \text{and}($   
~~~~ $\text{integer-is-less-or-equal}(M, \text{signed-bit-vector-maximum}(N)),$   
~~~~ $\text{integer-is-greater-or-equal}(M, \text{signed-bit-vector-minimum}(N))$

*Funcon*  $\text{is-in-unsigned-bit-vector}(M : \text{integers}, N : \text{natural-numbers}) : \Rightarrow \text{booleans}$   
~~~ $\rightsquigarrow \text{and}($   
~~~~ $\text{integer-is-less-or-equal}(M, \text{unsigned-bit-vector-maximum}(N)),$   
~~~~ $\text{integer-is-greater-or-equal}(M, 0)$