

Example Semantics for LOPSTR'13

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The semantics presented here originally appeared in Churchill and Mosses' *Modular Bisimulation Theory for Computations and Values* [2]. It has been adapted to be value-computation free; i.e., all value terms are considered fully normalized.

For details on how the refocused and striding rules were generated from the small-step semantics in Sect. 1 cf. [1].

Disclaimer: all rules in this file have been auto-generated by the MSOS Derivation Tool, which is still at an early development stage.

1 Small-Step Semantics

1.1 apply

$$\frac{\frac{\frac{\text{apply}(\text{v}(\text{abs}(x_1, \text{v}(x_2), x_3)), \text{v}(x_4)) \xrightarrow{\{-\}} \text{v}(x_2)}{\text{map_update}(x_1, x_2, \text{v}(x_3), x_4)} \quad x_5 \xrightarrow{\{\text{env}=x_4, X_1\}} x'_5}{\text{apply}(\text{v}(\text{abs}(x_2, x_5, x_1)), \text{v}(x_3)) \xrightarrow{\{\text{env}=x_6, X_1\}} \text{apply}(\text{v}(\text{abs}(x_2, x'_5, x_1)), \text{v}(x_3))}}{\frac{x_1 \xrightarrow{L_1} x'_1}{\text{apply}(\text{v}(x_2), x_1) \xrightarrow{L_1} \text{apply}(\text{v}(x_2), x'_1)}}}$$
$$\frac{x_1 \xrightarrow{L_1} x'_1}{\text{apply}(x_1, x_2) \xrightarrow{L_1} \text{apply}(x'_1, x_2)}$$

1.2 assign

$$\frac{\frac{x_1 \xrightarrow{L_1} x'_1}{\text{assign}(x_2, x_1) \xrightarrow{L_1} \text{assign}(x_2, x'_1)}}{\frac{\text{map_update}(x_1, x_2, \text{v}(x_3), x_4)}{\text{assign}(x_2, \text{v}(x_3)) \xrightarrow{\{\text{store}=x_1, \text{store}'=x_4, -\}} \text{v}(\text{skip})}}$$

1.3 atomic

$$\frac{\frac{\text{atomic}(\text{v}(x_1)) \xrightarrow{\{-\}} \text{v}(x_1)}{x_1 \xrightarrow{L_1} x'_1 \quad \text{atomic}(x'_1) \xrightarrow{L_2} \text{v}(x_2)}}{\text{atomic}(x_1) \xrightarrow{L_2 \circ L_1} \text{v}(x_2)}$$

1.4 bound

$$\frac{}{\text{bound}(x_2) \xrightarrow{\{\text{env}=x_1, -\}} \text{lookup}(x_1, x_2)}$$

1.5 catch

$$\begin{array}{c}
\frac{}{\text{catch}(v(x_1), x_2) \xrightarrow{\{\text{exc}'=v(\text{nil}), -\}} v(x_1)} \\
\frac{x_1 \xrightarrow{\{\text{exc}'=v(\text{cons}(x_2, v(\text{nil}))), \varepsilon=0, \varepsilon'=1, X_1\}} x'_1 \quad x_2 \neq v(\text{nil})}{\text{catch}(x_1, x_3) \xrightarrow{\{\text{exc}'=v(\text{nil}), \varepsilon=0, \varepsilon'=0, X_1\}} \text{apply}(x_3, x_2)} \\
\frac{x_1 \xrightarrow{\{\text{exc}'=v(\text{nil}), \varepsilon=0, \varepsilon'=0, X_1\}} x'_1}{\text{catch}(x_1, x_2) \xrightarrow{\{\text{exc}'=v(\text{nil}), \varepsilon=0, \varepsilon'=0, X_1\}} \text{catch}(x'_1, x_2)}
\end{array}$$

1.6 deref

$$\frac{}{\text{deref}(x_2) \xrightarrow{\{\text{store}=x_1, -\}} \text{lookup}(x_1, x_2)}$$

1.7 eq

$$\begin{array}{c}
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{eq}(x_2, x_1) \xrightarrow{L_1} \text{eq}(x_2, x'_1)} \\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{eq}(x_1, x_2) \xrightarrow{L_1} \text{eq}(x'_1, x_2)} \\
\frac{x_1 \neq x_2}{\text{eq}(v(x_1), v(x_2)) \xrightarrow{\{-\}} v(\text{false})} \\
\frac{x_1 = x_2}{\text{eq}(v(x_1), v(x_2)) \xrightarrow{\{-\}} v(\text{true})}
\end{array}$$

1.8 if

$$\begin{array}{c}
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{if}(x_1, x_2, x_3) \xrightarrow{L_1} \text{if}(x'_1, x_2, x_3)} \\
\frac{}{\text{if}(v(\text{false}), x_1, x_2) \xrightarrow{\{-\}} x_2} \\
\frac{}{\text{if}(v(\text{true}), x_1, x_2) \xrightarrow{\{-\}} x_1}
\end{array}$$

1.9 int_add

$$\begin{array}{c}
\frac{\text{msos_int_add}(x_1, x_2, x_3)}{\text{int_add}(v(x_1), v(x_2)) \xrightarrow{\{-\}} v(x_3)} \\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{int_add}(x_2, x_1) \xrightarrow{L_1} \text{int_add}(x_2, x'_1)} \\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{int_add}(x_1, x_2) \xrightarrow{L_1} \text{int_add}(x'_1, x_2)}
\end{array}$$

1.10 int_mod

$$\begin{array}{c}
\frac{\text{msos_int_mod}(x_1, x_2, x_3)}{\text{int_mod}(\mathbf{v}(x_1), \mathbf{v}(x_2)) \xrightarrow{\{-\}} \mathbf{v}(x_3)} \\
\\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{int_mod}(x_2, x_1) \xrightarrow{L_1} \text{int_mod}(x_2, x'_1)} \\
\\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{int_mod}(x_1, x_2) \xrightarrow{L_1} \text{int_mod}(x'_1, x_2)}
\end{array}$$

1.11 int_mul

$$\begin{array}{c}
\frac{\text{msos_int_mul}(x_1, x_2, x_3)}{\text{int_mul}(\mathbf{v}(x_1), \mathbf{v}(x_2)) \xrightarrow{\{-\}} \mathbf{v}(x_3)} \\
\\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{int_mul}(x_2, x_1) \xrightarrow{L_1} \text{int_mul}(x_2, x'_1)} \\
\\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{int_mul}(x_1, x_2) \xrightarrow{L_1} \text{int_mul}(x'_1, x_2)}
\end{array}$$

1.12 int_sub

$$\begin{array}{c}
\frac{\text{msos_int_sub}(x_1, x_2, x_3)}{\text{int_sub}(\mathbf{v}(x_1), \mathbf{v}(x_2)) \xrightarrow{\{-\}} \mathbf{v}(x_3)} \\
\\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{int_sub}(x_2, x_1) \xrightarrow{L_1} \text{int_sub}(x_2, x'_1)} \\
\\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{int_sub}(x_1, x_2) \xrightarrow{L_1} \text{int_sub}(x'_1, x_2)}
\end{array}$$

1.13 lambda

$$\frac{}{\text{lambda}(x_1, x_2) \xrightarrow{\{\text{env}=x_3, -\}} \mathbf{v}(\text{abs}(x_1, x_2, x_3))}$$

1.14 let

$$\begin{array}{c}
\frac{}{\text{let}(x_1, x_2, \mathbf{v}(x_3)) \xrightarrow{\{-\}} \mathbf{v}(x_3)} \\
\\
\frac{\text{map_update}(x_1, x_2, \mathbf{v}(x_3), x_4) \quad x_5 \xrightarrow{\{\text{env}=x_4, X_1\}} x'_5}{\text{let}(x_2, \mathbf{v}(x_3), x_5) \xrightarrow{\{\text{env}=x_1, X_1\}} \text{let}(x_2, \mathbf{v}(x_3), x'_5)} \\
\\
\frac{x_1 \xrightarrow{L_1} x'_1}{\text{let}(x_2, x_1, x_3) \xrightarrow{L_1} \text{let}(x_2, x'_1, x_3)}
\end{array}$$

1.15 lookup

$$\begin{array}{c}
\frac{}{\text{lookup}(\text{map_prefix}(x_1, \mathbf{v}(x_2), x_3), x_1) \xrightarrow{\{-\}} \mathbf{v}(x_2)} \\
\\
\frac{\text{lookup}(x_1, x_2) \xrightarrow{\{-\}} \mathbf{v}(x_3) \quad x_4 \neq x_2}{\text{lookup}(\text{map_prefix}(x_4, \mathbf{v}(x_5), x_1), x_2) \xrightarrow{\{-\}} \mathbf{v}(x_3)}
\end{array}$$

1.16 print

$$\text{print}(x_1) \xrightarrow{\{\text{output}'=x_1, \text{---}\}} \text{v}(\text{skip})$$

1.17 seq

$$\begin{array}{c} \xrightarrow{\{\text{---}\}} \text{seq}(\text{v}(\text{skip}), x_1) \xrightarrow{\{\text{---}\}} x_1 \\ \hline x_1 \xrightarrow{L_1} x'_1 \\ \hline \text{seq}(x_1, x_2) \xrightarrow{L_1} \text{seq}(x'_1, x_2) \end{array}$$

1.18 throw

$$\text{throw}(x_1) \xrightarrow{\{\text{exc}'=\text{v}(\text{cons}(x_1, \text{v}(\text{nil}))), \varepsilon'=1, \text{---}\}} \text{s_}(\text{stuck})$$

1.19 while

$$\text{while}(x_1, x_2) \xrightarrow{\{\text{---}\}} \text{if}(x_1, \text{seq}(x_2, \text{while}(x_1, x_2)), \text{v}(\text{skip}))$$

2 Refocused Semantics

2.1 apply

$$\begin{array}{c}
\frac{}{\text{apply}(\text{v}(\text{abs}(x_1, \text{v}(x_2), x_3)), \text{v}(x_4)) \xrightarrow{\{-\}} \text{v}(x_2)} \\
\frac{\text{map_update}(x_2, x_3, \text{v}(x_4), x_5) \quad x_6 \xrightarrow{\{\text{env}=x_5, X_1\}} x'_6 \quad \text{apply}(\text{v}(\text{abs}(x_3, x'_6, x_2)), \text{v}(x_4)) \xrightarrow{L_2}^* x_7}{\text{apply}(\text{v}(\text{abs}(x_3, x_6, x_2)), \text{v}(x_4)) \xrightarrow{L_2 \circ \{\varepsilon=0, \text{env}=x_1, X_1\}} x_7} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{apply}(\text{v}(x_2), x'_1) \xrightarrow{L_2}^* x_3}{\text{apply}(\text{v}(x_2), x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{apply}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{apply}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

2.2 assign

$$\begin{array}{c}
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{assign}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{assign}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{\text{map_update}(x_1, x_2, \text{v}(x_3), x_4)}{\text{assign}(x_2, \text{v}(x_3)) \xrightarrow{\{\text{store}=x_1, \text{store}'=x_4, -\}} \text{v}(\text{skip})}
\end{array}$$

2.3 atomic

$$\begin{array}{c}
\frac{}{\text{atomic}(\text{v}(x_1)) \xrightarrow{\{-\}} \text{v}(x_1)} \\
\frac{x_1 \xrightarrow{L_2} x'_1 \quad \text{atomic}(x'_1) \xrightarrow{L_3} \text{v}(x_2)}{\text{atomic}(x_1) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\}} \text{v}(x_2)}
\end{array}$$

2.4 bound

$$\frac{}{\text{bound}(x_2) \xrightarrow{\{\text{env}=x_1, -\}} \text{lookup}(x_1, x_2)}$$

2.5 catch

$$\begin{array}{c}
\frac{}{\text{catch}(\text{v}(x_1), x_2) \xrightarrow{\{\text{exc}'=\text{v}(\text{nil}), -\}} \text{v}(x_1)} \\
\frac{x_1 \xrightarrow{\{\text{exc}'=\text{v}(\text{cons}(x_2, \text{v}(\text{nil}))), \varepsilon=0, \varepsilon'=1, X_1\}} x'_1 \quad x_2 \neq \text{v}(\text{nil}) \quad \text{apply}(x_3, x_2) \xrightarrow{L_2}^* x_4}{\text{catch}(x_1, x_3) \xrightarrow{L_2 \circ \{\varepsilon=0, \text{exc}'=\text{v}(\text{nil}), \varepsilon'=0, X_1\}} x_4} \\
\frac{x_1 \xrightarrow{\{\text{exc}'=\text{v}(\text{nil}), \varepsilon=0, \varepsilon'=0, X_1\}} x'_1 \quad \text{catch}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{catch}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, \text{exc}'=\text{v}(\text{nil}), \varepsilon'=0, X_1\}} x_3}
\end{array}$$

2.6 deref

$$\frac{}{\text{deref}(x_2) \xrightarrow{\{\text{store}=x_1, -\}} \text{lookup}(x_1, x_2)}$$

2.7 eq

$$\begin{array}{c}
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{eq}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{eq}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{eq}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{eq}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \neq x_2}{\text{eq}(v(x_1), v(x_2)) \xrightarrow{\{-\}} v(\text{false})} \\
\frac{x_1 = x_2}{\text{eq}(v(x_1), v(x_2)) \xrightarrow{\{-\}} v(\text{true})}
\end{array}$$

2.8 if

$$\begin{array}{c}
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{if}(x'_1, x_2, x_3) \xrightarrow{L_2}^* x_4}{\text{if}(x_1, x_2, x_3) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_4} \\
\frac{}{\text{if}(v(\text{false}), x_1, x_2) \xrightarrow{\{-\}} x_2} \\
\frac{}{\text{if}(v(\text{true}), x_1, x_2) \xrightarrow{\{-\}} x_1}
\end{array}$$

2.9 int_add

$$\begin{array}{c}
\frac{\text{msos_int_add}(x_1, x_2, x_3)}{\text{int_add}(v(x_1), v(x_2)) \xrightarrow{\{-\}} v(x_3)} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_add}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_add}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_add}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{int_add}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

2.10 int_mod

$$\begin{array}{c}
\frac{\text{msos_int_mod}(x_1, x_2, x_3)}{\text{int_mod}(v(x_1), v(x_2)) \xrightarrow{\{-\}} v(x_3)} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_mod}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_mod}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_mod}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{int_mod}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

2.11 int_mul

$$\begin{array}{c}
\frac{\text{msos_int_mul}(x_1, x_2, x_3)}{\text{int_mul}(\mathbf{v}(x_1), \mathbf{v}(x_2)) \xrightarrow{\{-\}} \mathbf{v}(x_3)} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_mul}(x_2, x'_1) \xrightarrow{L_2, *} x_3}{\text{int_mul}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_mul}(x'_1, x_2) \xrightarrow{L_2, *} x_3}{\text{int_mul}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

2.12 int_sub

$$\begin{array}{c}
\frac{\text{msos_int_sub}(x_1, x_2, x_3)}{\text{int_sub}(\mathbf{v}(x_1), \mathbf{v}(x_2)) \xrightarrow{\{-\}} \mathbf{v}(x_3)} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_sub}(x_2, x'_1) \xrightarrow{L_2, *} x_3}{\text{int_sub}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_sub}(x'_1, x_2) \xrightarrow{L_2, *} x_3}{\text{int_sub}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

2.13 lambda

$$\frac{}{\text{lambda}(x_1, x_2) \xrightarrow{\{\text{env}=x_3, -\}} \mathbf{v}(\text{abs}(x_1, x_2, x_3))}$$

2.14 let

$$\begin{array}{c}
\frac{}{\text{let}(x_1, x_2, \mathbf{v}(x_3)) \xrightarrow{\{-\}} \mathbf{v}(x_3)} \\
\frac{\text{map_update}(x_1, x_2, \mathbf{v}(x_3), x_4) \quad x_5 \xrightarrow{\{\text{env}=x_4, X_1\}} x'_5 \quad \text{let}(x_2, \mathbf{v}(x_3), x'_5) \xrightarrow{L_2, *} x_6}{\text{let}(x_2, \mathbf{v}(x_3), x_5) \xrightarrow{L_2 \circ \{\varepsilon=0, \text{env}=x_1, X_1\}} x_6} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{let}(x_2, x'_1, x_3) \xrightarrow{L_2, *} x_4}{\text{let}(x_2, x_1, x_3) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_4}
\end{array}$$

2.15 lookup

$$\begin{array}{c}
\frac{}{\text{lookup}(\text{map_prefix}(x_1, \mathbf{v}(x_2), x_3), x_1) \xrightarrow{\{-\}} \mathbf{v}(x_2)} \\
\frac{\text{lookup}(x_1, x_2) \xrightarrow{\{\varepsilon=0, -\}} \mathbf{v}(x_3) \quad x_4 \neq x_2}{\text{lookup}(\text{map_prefix}(x_4, \mathbf{v}(x_5), x_1), x_2) \xrightarrow{\{-\} \circ \{\varepsilon=0, -\}} \mathbf{v}(x_3)}
\end{array}$$

2.16 print

$$\frac{}{\text{print}(x_1) \xrightarrow{\{\text{output}'=x_1, -\}} \mathbf{v}(\text{skip})}$$

2.17 seq

$$\frac{\frac{\text{seq}(\text{v}(\text{skip}), x_1) \xrightarrow{\{-\}} x_1}{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1} \quad \text{seq}(x'_1, x_2) \xrightarrow{L_2^*} x_3}{\text{seq}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}$$

2.18 throw

$$\text{throw}(x_1) \xrightarrow{\{\text{exc}'=\text{v}(\text{cons}(x_1, \text{v}(\text{nil}))), \varepsilon'=1, -\}} \text{s}_-(\text{stuck})$$

2.19 while

$$\text{while}(x_1, x_2) \xrightarrow{\{-\}} \text{if}(x_1, \text{seq}(x_2, \text{while}(x_1, x_2)), \text{v}(\text{skip}))$$

3 Striding Semantics

3.1 apply

$$\begin{array}{c}
\frac{x_2 \xrightarrow{\{\varepsilon=0, X_3\}}^* \mathbf{v}(\mathbf{abs}(x_3, x_4, x_5)) \quad x_6 \xrightarrow{\{\varepsilon=0, X_2\}}^* \mathbf{v}(x_7) \quad \mathbf{map_update}(x_5, x_3, \mathbf{v}(x_7), x_8) \quad x_4 \xrightarrow{\{\mathbf{env}=x_8, X_1\}}^* \mathbf{v}(x_9)}{\mathbf{apply}(x_2, x_6) \xrightarrow{\{_ \} \circ \{\varepsilon=0, \mathbf{env}=x_1, X_1\} \circ \{\varepsilon=0, X_2\} \circ \{\varepsilon=0, X_3\}} \mathbf{v}(x_9)} \\
\\
\frac{x_2 \xrightarrow{\{\varepsilon=0, X_4\}}^* \mathbf{v}(\mathbf{abs}(x_3, x_4, x_5)) \quad x_6 \xrightarrow{\{\varepsilon=0, X_3\}}^* \mathbf{v}(x_7) \quad \mathbf{map_update}(x_5, x_3, \mathbf{v}(x_7), x_8) \quad x_4 \xrightarrow{\{\mathbf{env}=x_8, X_1\}} x_9}{\mathbf{apply}(\mathbf{v}(\mathbf{abs}(x_3, x_9, x_5)), \mathbf{v}(x_7)) \xrightarrow{L_2}^* x_{10}} \\
\hline
\mathbf{apply}(x_2, x_6) \xrightarrow{L_2 \circ \{\varepsilon=0, \mathbf{env}=x_1, X_1\} \circ \{\varepsilon=0, X_3\} \circ \{\varepsilon=0, X_4\}} x_{10} \\
\\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* \mathbf{v}(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}} x'_3 \quad \mathbf{apply}(\mathbf{v}(x_2), x'_3) \xrightarrow{L_2}^* x_4}{\mathbf{apply}(x_1, x_3) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_4} \\
\\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \mathbf{apply}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\mathbf{apply}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

3.2 assign

$$\begin{array}{c}
\frac{x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* \mathbf{v}(x_4) \quad \mathbf{map_update}(x_1, x_5, \mathbf{v}(x_4), x_2)}{\mathbf{assign}(x_5, x_3) \xrightarrow{\{\mathbf{store}=x_1, \mathbf{store}'=x_2, _ \} \circ \{\varepsilon=0, X_1\}} \mathbf{v}(\mathbf{skip})} \\
\\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \mathbf{assign}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\mathbf{assign}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

3.3 atomic

$$\begin{array}{c}
\frac{}{\mathbf{atomic}(\mathbf{v}(x_1)) \xrightarrow{\{_ \}} \mathbf{v}(x_1)} \\
\\
\frac{x_1 \xrightarrow{L_2} x'_1 \quad \mathbf{atomic}(x'_1) \xrightarrow{L_3} \mathbf{v}(x_2)}{\mathbf{atomic}(x_1) \xrightarrow{\{_ \} \circ \{\varepsilon=0, X_1\}} \mathbf{v}(x_2)}
\end{array}$$

3.4 bound

$$\begin{array}{c}
\frac{}{\mathbf{bound}(x_1) \xrightarrow{\{_ \} \circ \{\varepsilon=0, \mathbf{env}=\mathbf{map_prefix}(x_1, \mathbf{v}(x_2), x_3), _ \}} \mathbf{v}(x_2)} \\
\\
\frac{\mathbf{lookup}(x_3, x_4) \xrightarrow{\{\varepsilon=0, _ \}} \mathbf{v}(x_5) \quad x_1 \neq x_4}{\mathbf{bound}(x_4) \xrightarrow{\{_ \} \circ \{\varepsilon=0, _ \} \circ \{\varepsilon=0, \mathbf{env}=\mathbf{map_prefix}(x_1, \mathbf{v}(x_2), x_3), _ \}} \mathbf{v}(x_5)}
\end{array}$$

3.5 catch

$$\begin{array}{c}
\frac{x_1 \xrightarrow{\{\varepsilon=0, \varepsilon'=0, \mathbf{exc}'=\mathbf{v}(\mathbf{nil}), X_1\}}^* \mathbf{v}(x_2)}{\mathbf{catch}(x_1, x_3) \xrightarrow{\{\varepsilon=0, \varepsilon'=0, \mathbf{exc}'=\mathbf{v}(\mathbf{nil}), _ \} \circ \{\varepsilon=0, \varepsilon'=0, \mathbf{exc}'=\mathbf{v}(\mathbf{nil}), X_1\}} \mathbf{v}(x_2)} \\
\\
\frac{x_1 \xrightarrow{\{\varepsilon=0, \varepsilon'=1, \mathbf{exc}'=\mathbf{v}(\mathbf{cons}(x_2, \mathbf{v}(\mathbf{nil}))), X_1\}} x'_1 \quad x_2 \neq \mathbf{v}(\mathbf{nil}) \quad \mathbf{apply}(x_3, x_2) \xrightarrow{L_2}^* x_4}{\mathbf{catch}(x_1, x_3) \xrightarrow{L_2 \circ \{\varepsilon=0, \varepsilon'=0, \mathbf{exc}'=\mathbf{v}(\mathbf{nil}), X_1\}} x_4} \\
\\
\frac{x_1 \xrightarrow{\{\varepsilon=0, \varepsilon'=0, \mathbf{exc}'=\mathbf{v}(\mathbf{nil}), X_1\}} x'_1 \quad \mathbf{catch}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\mathbf{catch}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, \varepsilon'=0, \mathbf{exc}'=\mathbf{v}(\mathbf{nil}), X_1\}} x_3}
\end{array}$$

3.6 deref

$$\text{deref}(x_2) \xrightarrow{\{\text{store}=x_1, -\}} \text{lookup}(x_1, x_2)$$

3.7 eq

$$\begin{array}{c} \frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad x_4 \neq x_2}{\text{eq}(x_3, x_1) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(\text{false})} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad x_4 = x_2}{\text{eq}(x_3, x_1) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(\text{true})} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad x_2 \neq x_4}{\text{eq}(x_1, x_3) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(\text{false})} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad x_2 = x_4}{\text{eq}(x_1, x_3) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(\text{true})} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{eq}(x'_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{eq}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{eq}(x'_1, x'_2) \xrightarrow{L_2}^* x_3}{\text{eq}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{eq}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{eq}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{eq}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{eq}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \end{array}$$

3.8 if

$$\begin{array}{c} \frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(\text{false})}{\text{if}(x_1, x_2, x_3) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\}} x_3} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(\text{true})}{\text{if}(x_1, x_2, x_3) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\}} x_2} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{if}(x'_1, x_2, x_3) \xrightarrow{L_2}^* x_4}{\text{if}(x_1, x_2, x_3) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_4} \end{array}$$

3.9 int_add

$$\begin{array}{c} \frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad \text{msos_int_add}(x_4, x_2, x_5)}{\text{int_add}(x_3, x_1) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_5)} \\ \frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad \text{msos_int_add}(x_2, x_4, x_5)}{\text{int_add}(x_1, x_3) \xrightarrow{\{-\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_5)} \end{array}$$

$$\begin{array}{c}
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{int_add}(x'_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_add}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{int_add}(x'_1, x'_2) \xrightarrow{L_2}^* x_3}{\text{int_add}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_add}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_add}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_add}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{int_add}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

3.10 int_mod

$$\begin{array}{c}
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad \text{msos_int_mod}(x_4, x_2, x_5)}{\text{int_mod}(x_3, x_1) \xrightarrow{\{\text{---}\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_5)} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad \text{msos_int_mod}(x_2, x_4, x_5)}{\text{int_mod}(x_1, x_3) \xrightarrow{\{\text{---}\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_5)} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{int_mod}(x'_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_mod}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{int_mod}(x'_1, x'_2) \xrightarrow{L_2}^* x_3}{\text{int_mod}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_mod}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_mod}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_mod}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{int_mod}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

3.11 int_mul

$$\begin{array}{c}
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad \text{msos_int_mul}(x_4, x_2, x_5)}{\text{int_mul}(x_3, x_1) \xrightarrow{\{\text{---}\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_5)} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad \text{msos_int_mul}(x_2, x_4, x_5)}{\text{int_mul}(x_1, x_3) \xrightarrow{\{\text{---}\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_5)} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{int_mul}(x'_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_mul}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{int_mul}(x'_1, x'_2) \xrightarrow{L_2}^* x_3}{\text{int_mul}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3} \\
\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_mul}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_mul}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}
\end{array}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_mul}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{int_mul}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}$$

3.12 int_sub

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad \text{msos_int_sub}(x_4, x_2, x_5)}{\text{int_sub}(x_3, x_1) \xrightarrow{\{\text{---}\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_5)}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_2) \quad x_3 \xrightarrow{\{\varepsilon=0, X_1\}}^* v(x_4) \quad \text{msos_int_sub}(x_2, x_4, x_5)}{\text{int_sub}(x_1, x_3) \xrightarrow{\{\text{---}\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_5)}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{int_sub}(x'_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_sub}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_3\}}^* x'_1 \quad x_2 \xrightarrow{\{\varepsilon=0, X_1\}} x'_2 \quad \text{int_sub}(x'_1, x'_2) \xrightarrow{L_2}^* x_3}{\text{int_sub}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, X_3\}} x_3}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_sub}(x_2, x'_1) \xrightarrow{L_2}^* x_3}{\text{int_sub}(x_2, x_1) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{int_sub}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\text{int_sub}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}$$

3.13 lambda

$$\frac{}{\text{lambda}(x_1, x_2) \xrightarrow{\{\text{env}=x_3, \text{---}\}} v(\text{abs}(x_1, x_2, x_3))}$$

3.14 let

$$\frac{x_2 \xrightarrow{\{\varepsilon=0, X_2\}}^* v(x_3) \quad \text{map_update}(x_1, x_4, v(x_3), x_5) \quad x_6 \xrightarrow{\{\text{env}=x_5, X_1\}}^* v(x_7)}{\text{let}(x_4, x_2, x_6) \xrightarrow{\{\text{---}\} \circ \{\varepsilon=0, \text{env}=x_1, X_1\} \circ \{\varepsilon=0, X_2\}} v(x_7)}$$

$$\frac{x_2 \xrightarrow{\{\varepsilon=0, X_3\}}^* v(x_3) \quad \text{map_update}(x_1, x_4, v(x_3), x_5) \quad x_6 \xrightarrow{\{\text{env}=x_5, X_1\}} x'_6 \quad \text{let}(x_4, v(x_3), x'_6) \xrightarrow{L_2}^* x_7}{\text{let}(x_4, x_2, x_6) \xrightarrow{L_2 \circ \{\varepsilon=0, \text{env}=x_1, X_1\} \circ \{\varepsilon=0, X_3\}} x_7}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \text{let}(x_2, x'_1, x_3) \xrightarrow{L_2}^* x_4}{\text{let}(x_2, x_1, x_3) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_4}$$

3.15 lookup

$$\frac{}{\text{lookup}(\text{map_prefix}(x_1, v(x_2), x_3), x_1) \xrightarrow{\{\text{---}\}} v(x_2)}$$

$$\frac{\text{lookup}(x_1, x_2) \xrightarrow{\{\varepsilon=0, \text{---}\}} v(x_3) \quad x_4 \neq x_2}{\text{lookup}(\text{map_prefix}(x_4, v(x_5), x_1), x_2) \xrightarrow{\{\text{---}\} \circ \{\varepsilon=0, \text{---}\}} v(x_3)}$$

3.16 print

$$\frac{}{\text{print}(x_1) \xrightarrow{\{\text{output}'=x_1, \text{---}\}} v(\text{skip})}$$

3.17 seq

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}}^* \mathbf{v}(\mathbf{skip})}{\mathbf{seq}(x_1, x_2) \xrightarrow{\{—\} \circ \{\varepsilon=0, X_1\}} x_2}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \mathbf{seq}(x'_1, x_2) \xrightarrow{L_2}^* x_3}{\mathbf{seq}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\}} x_3}$$

3.18 throw

$$\mathbf{throw}(x_1) \xrightarrow{\{\mathbf{exc}'=\mathbf{v}(\mathbf{cons}(x_1, \mathbf{v}(\mathbf{nil}))), \varepsilon'=1, —\}} \mathbf{s_stuck}$$

3.19 while

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}}^* \mathbf{v}(\mathbf{false})}{\mathbf{while}(x_1, x_2) \xrightarrow{\{—\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, —\}} \mathbf{v}(\mathbf{skip})}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}}^* \mathbf{v}(\mathbf{true})}{\mathbf{while}(x_1, x_2) \xrightarrow{\{—\} \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, —\}} \mathbf{seq}(x_2, \mathbf{while}(x_1, x_2))}$$

$$\frac{x_1 \xrightarrow{\{\varepsilon=0, X_1\}} x'_1 \quad \mathbf{if}(x'_1, \mathbf{seq}(x_2, \mathbf{while}(x_1, x_2)), \mathbf{v}(\mathbf{skip})) \xrightarrow{L_2}^* x_3}{\mathbf{while}(x_1, x_2) \xrightarrow{L_2 \circ \{\varepsilon=0, X_1\} \circ \{\varepsilon=0, —\}} x_3}$$

References

- [1] Casper Bach Poulsen and Peter D. Mosses. “Generating Specialized Interpreters for Modular Structural Operational Semantics”. In: *LOPSTR'13*. Springer, 2013.
- [2] Martin Churchill and Peter D. Mosses. “Modular Bisimulation Theory for Computations and Values”. In: *FOSSACS'13*. Vol. 7794. LNCS. Springer, 2013, pp. 97–112.